ARE 202B
Teddy Wong/Aya Suzuki/Navin Yavapolkul

# Fertility rate growth and Becker HH model

## 1. Introduction

Resource constraints often make population growth a major issue for developing countries. Likewise, declining population growth resulting from a diminishing fertility rate is a serious concern for developed countries. In both cases, fertility rate is an important factor for healthy economic growth and stability. This short paper, which uses Gary Becker's Household Production Model as the starting point, tries to explain the factors behind the shift in fertility rate as societies develop.

## 2. The Model

We consider a household that derives utility from goods $z_i$, (for i=1,…,I) and good $z_k$, which represent the number of children (k for kids). Using Becker's Household Model, a household's utility function and transformation functions are as follows:

$U(z_i, z_k)$ for i = 1, … ,I and k ; where $z_k$ represents children in the household.

The transformation function are:

$z_i \leq f_i(x_i, t_i)$ for all i , and $z_k \leq f_k(x_k, t_k)$

The $\boldsymbol{x}$'s represent market goods that are purchased and $\boldsymbol{t}$'s represent time spent producing the $\boldsymbol{z}$ good in the household.

A household is endowed with time $\boldsymbol{T}$, which is divided into time spent for producing children $\boldsymbol{t_k}$, and time spent producing good I, expressed as $\boldsymbol{t_i}$. The child contributes to the household's budget constraint in some developing countries that allow for child labor. We assume that the child produces labor $\boldsymbol{l}$, which is an increasing function of $\boldsymbol{z_k}$, $l = \psi(z_k)$. This function is not being determined through a maximization process and so is exogenously determined by the parents who can demand the amount of labor each child provides for the household. In other words, child labor is not a function of child wage or adult wage. Also, the child labor wage $\boldsymbol{w_2}$ is allowed to be different from the adult wage $\boldsymbol{w_1}$. Then the maximization problem of the household is given by:

**Max** $U(z_i, z_k)$

**Subject to:**

$$\sum p_i x_i + p_k x_k + w_1 \sum t_i + w_1 t_k \leq \overline{Y} + T w_1 + w_2 l$$

$$z_k \leq f_k(x_k, t_k) \quad , \quad l = \psi(z_k)$$

$$z_i \leq f_i(x_i, t_i) \quad \forall i = 1, \ldots, I$$

Plus the usual non-negativity constraints

**The Lagrangian is given by:**

$$L = U(z_i, z_k) + \lambda(\overline{Y} + T w_1 + w_2 l - \sum p_i x_i - p_k x_k - w_1 \sum t_i - w_1 t_k) + \sum \mu_i (f_i(x_i, t_i) - z_i) + \mu_k (f_k(x_k, t_k) - z_k)$$

**The FONCs are:**

**(1)** $\frac{\partial L}{\partial z_k} = \frac{\partial U}{\partial z_k} + \lambda w_2 \psi'(z_k) - \mu_k = 0$

**(2)** $\frac{\partial L}{\partial z_i} = \frac{\partial U}{\partial z_i} - \mu_i = 0 \quad \forall i = 1, \ldots, I$

**(3)** $\frac{\partial L}{\partial x_k} = -\lambda p_k + \mu_k \frac{\partial f_k}{\partial x_k} = 0 \quad \forall k = 1, \ldots, K \quad \textit{where} \quad K \leq I$

**(4)** $\frac{\partial L}{\partial x_i} = -\lambda p_i + \mu_i \frac{\partial f_i}{\partial x_i} = 0 \quad \forall i = 1, \ldots, I$

**(5)** $\frac{\partial L}{\partial t_k} = -\lambda w_1 + \mu_k \frac{\partial f_k}{\partial t_k}$

**(6)** $\frac{\partial L}{\partial t_i} = -\lambda w_1 + \mu_i \frac{\partial f_i}{\partial t_i} = 0 \quad \forall i = 1, \ldots, I$

Combining (3) and (5), we obtain $\frac{\partial f_k / \partial t_k}{\partial f_k / \partial x_k} = \frac{w_1}{p_k}$ **(7)**, which is the equality of the marginal products with the ratio of factor prices.

If we solve for in $\mu_k$ eqn (3) and (5) we obtain $\mu_k = \frac{\lambda p_k}{\partial f_k / \partial x_k} = \frac{\lambda w_1}{\partial f_k / \partial t_k}$ **(8)**

and solving for $\mu_k$ in (1) and substituting in equation (8) we obtain

$$\frac{\partial U}{\partial z_k} + \lambda w_2 \psi'(z_k) = \mu_k = \frac{\lambda p_k}{\partial f_k / \partial x_k} = \frac{\lambda w_1}{\partial f_k / \partial t_k} \quad \textbf{(9)}$$

We know that $\lambda = \frac{\partial U}{\partial Y}$, equating lambda with the marginal utility of relaxing our budget constraint. Thus,

$$\frac{\partial U}{\partial z_k} + \frac{\partial U}{\partial Y} w_2 \psi'(z_k) = \frac{\partial U / \partial Y}{\partial f_k / \partial t_k} w_1 \quad \textbf{(10)}$$

and

$$\frac{\partial U}{\partial z_k} + \frac{\partial U}{\partial Y} w_2 \psi'(z_k) = \frac{\partial U / \partial Y}{\partial f_k / \partial x_k} p_k \quad \textbf{(11)}$$

Which yields:

$$\frac{\partial U}{\partial z_k} = \left[ \frac{w_1}{\partial f_k / \partial t_k} - w_2 \psi'(z_k) \right] \frac{\partial U}{\partial Y} \quad \textbf{(12)}$$

and

$$\frac{\partial U}{\partial z_k} = \left[ \frac{P_k}{\partial f_k / \partial x_k} - w_2 \psi'(z_k) \right] \frac{\partial U}{\partial Y} \quad \textbf{(13)}$$

Solving for lambda in equation (3) and (5), we obtain;

$$\lambda = \frac{\mu_k \, \partial f_k / \partial x_k}{P_k} \quad \textbf{(14)}$$

and

$$\lambda = \frac{\mu_k \, \partial f_k / \partial t_k}{w_1} \quad \textbf{(15)}$$

Plug this back in place of $\frac{\partial U}{\partial Y}$ in equations (12) and (13),

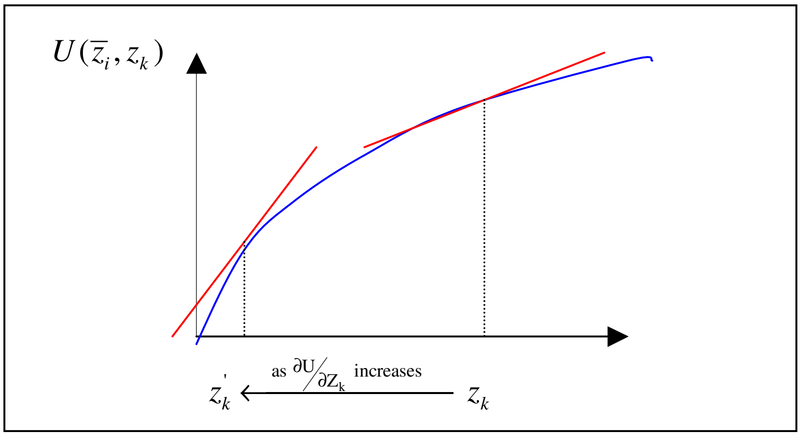
$$\frac{\partial U}{\partial z_k} = \mu_k \left[ 1 - \frac{w_2}{w_1} \frac{\partial f_k}{\partial t_k} \psi'(z_k) \right] \quad \textbf{(16)}$$

and

$$\frac{\partial U}{\partial z_k} = \mu_k \left[ 1 - \frac{w_2}{p_k} \frac{\partial f_k}{\partial x_k} \psi'(z_k) \right] \quad \textbf{(17)}$$

## 3. Observations

From Sanggon (2004), we know the effects of the marginal product of time and marginal product of inputs on the production of children in the household. Here, we will focus on the effect of child labor and other factors on the production of children in the household. If we assume a concave utility function such that it exhibits diminishing marginal utility of children, then a higher marginal utility of $z_k$ will indicate that a lower number of $z_k$ is being consumed.




***On the effect of child wage***

If we look at equation (12), we can see that as $w_2$, the child wage, gets larger, the marginal utility of $z_k$ will decrease, indicating an increase in the number of children that will be produced. If $w_2$ decreases, then the effect is the opposite – the household will choose to have fewer children. In this model, $w_2$ is positively correlated with $z_k$. Thus, when child labor is present, fluctuations in child labor wage may explain changes in the fertility rates.

***On the nonexistence of child labor market***

At the limit, when $w_2$ is zero, then effectively, the market for child labor disappears as is the case when countries develop and pass restrictions on child labor. We can see from equation (12), (13), (16), and (17) that if $w_2$ is zero, then the marginal utility of $z_k$ increases and there will be fewer children produced. This may explain the use of mandatory schooling and child labor laws as mechanisms of reducing fertility rate in developing countries, and this may also provide a reason as to why there exists a difference in fertility rates between developing and developed countries.

***On the effect of adult wage***

From equation (15) and (16), we can observe the relationship between the marginal utility of $z_k$ and adult wage. From (15), when adult wage $w_1$ increases, the marginal utility of $z_k$ increases, thus parents will choose fewer children. This is as expected and explains the lower fertility rate in developed countries compared to developing countries. In our model, the increase in $w_1$ relaxes the budget constraint, so the parents substitute away from $z_k$ since they can consume the same amount of goods and attain the same utility with fewer children contributing to the household budget. In other words, they are able to depend less on child labor for household consumption.

***Other effects on fertility***

$P_k$ is the price of inputs used to produce children. This could be the cost of food, education, or healthcare. From (17), we can see that as $p_k$ increases, the effect will be a reduction in number of children. On the other hand, the marginal labor of children $\psi'(z_k)$ also determines the level of fertility. If it were possible to reduce the marginal labor of children, the fertility rate would decline. The first effect, the price of inputs, may explain lower fertility rates in industrialized countries as the cost of raising children has increased substantially in recent decades. The second effect, the marginal labor of children, explains the relative contribution of children to the household budget in developed versus developing countries. The $\psi'(z_k)$ in developed countries may be smaller than the $\psi'(z_k)$ in developing countries, and this may be another reason that explains the deviation in fertility rates.

## 4. Conclusion

Because utility functions give us only ordinal relationships and the general nature of this model, we can only establish positive and negative relationships between optimal fertility rate and variables such as child labor wage, the existence of child labor markets, adult wages, and costs associated with raising children. However, there are many policy implications that may be conceived simply from knowing the sign of these effects. With more empirical work, it may be possible to impose functional forms for the utility function or transformation functions. Then the magnitudes of these effects may become clearer and the changes in fertility rate may be better understood, as the linkages between cause and effect will become more direct.